On logical and concurrent equivalences

J. C. Bradfield and S. B. Fröschle

Laboratory for Foundations of Computer Science, Division of Informatics, King's Bldgs, University of Edinburgh, Edinburgh EH9 3JZ, UK.
{jcb,sib}@dcs.ed.ac.uk

Abstract: We consider modal analogues of Hintikka et al.'s 'independence-friendly first-order logic', and discuss their relationship to equivalences previously studied in concurrency theory.

Key words: Independence, concurrency, branching quantifiers, modal logic.

1 Introduction

In [AHK97], Alur, Henzinger and Kupfermann introduced Alternating Temporal Logic, based on certain imperfect information games, in which independent 'teams' synchronize. In [Bra00], the first author proposed the application of logics based on Henkin quantifiers to modal logic in computer science; such logics include ATL, but also allow more powerful forms of expression. In that paper, we argued that making sense of such logics required some notion of locality in processes. After establishing some basic facts about such logics, we left open the obvious question of how such logics relate to established notions of independence and concurrency in computer science.

In this paper, we first interpret Henkin modal logics in a setting without locality (at least, without explicit locality), and then relate them to some of the natural true concurrent notions in the literature. The results here are preliminary, but, we believe, go some way towards a satisfactory explanation, and open up many further questions.

2 Henkin quantifiers and independence-friendly logic

We give a brief summary of the notions of Henkin quantifier and independence-friendly logic.

A *branching quantifier* Q is a set $\{x_1, \ldots, x_m, y_1, \ldots, y_n\}$ of variables, carrying a partial order $\prec$; the x_i are universal, the y_i existential. The semantics of $Q\phi$ is defined to be that of $\exists f_1 \ldots f_n . \forall x_1 \ldots x_m . \phi[f_i(y_i\downarrow)/y_i]$, where $y_i\downarrow$ is the list of variables $\prec y_i$, and $[\cdot/\cdot]$ denotes syntactic substitution: thus f_i is a Skolem function for y_i, but it refers only to variables preceding y_i in the partial order.

In particular, the *Henkin quantifier* $\begin{smallmatrix}\forall & \exists \\ \forall & \exists\end{smallmatrix} = \{x_1, x_2, y_1, y_2\}$ with $x_i \prec y_i$ is written $\begin{smallmatrix}\forall x_1 & \exists y_1 \\ \forall x_2 & \exists y_2\end{smallmatrix}$; thus $\begin{smallmatrix}\forall x & \exists y \\ \forall u & \exists v\end{smallmatrix}\phi(x, y, u, v)$ is equivalent by definition to $\exists f, g. \forall x, u. \phi(x, f(x), u, g(u))$.

Henkin quantifiers turn out to have existential second-order power, and are thus a strong operator to add to one's logic.

An alternative way of giving semantics to branching quantifiers is via games. Recall the Hintikka model-checking game for first-order logic (in positive form): given a formula ψ and a structure M, a position is a subformula $\phi(\vec{x})$ of ψ together with a *deal* for ϕ, that is, an assignment of values $\vec{v}$ to its free variables $\vec{x}$. At a position $(\forall x. \phi_1, \vec{v})$, Abelard chooses a value v for x, and play moves to the position $(\phi_1, \vec{v} \cdot v)$; similarly Eloise moves at $\exists x. \phi$. At $(\phi_1 \wedge \phi_2, \vec{v})$, Abelard chooses a conjunct ϕ_i, and play moves to $(\phi_i(\vec{x}'), \vec{v}')$, where $\vec{x}', \vec{v}'$ are $\vec{x}, \vec{v}$ restricted to the free variables of ϕ_i; and at $(\phi_1 \vee \phi_2, \vec{v})$, Eloise similarly chooses a disjunct. A play of the game terminates at (negated) atoms $(P(\vec{x}), \vec{v})$ (resp. $(\neg P(\vec{x}), \vec{v})$), and is won by Eloise (resp. Abelard) iff $P(\vec{v})$ is true. Then it is standard that $M \vDash \phi$ exactly if Eloise has a winning strategy in this game, where a strategy is a function from sequences of legal positions to moves.

These games have *perfect information*; both players know everything that has happened, and in particular when one player makes a choice, they know the other player's previous choices. Game semantics for the Henkin quantifiers, following [HiS96], use games of imperfect information: in the game for $\begin{smallmatrix}\forall x & \exists y \\ \forall u & \exists v\end{smallmatrix}\phi$, when Eloise chooses for v, she does not know what Abelard chose for x. To make this explicit, the logic is written with a more general syntax which is linear rather than two dimensional. A full account of the appropriate logic requires several new constructs, some of which raise subtle issues [Hod97]; we shall work with a restricted version which is sufficient to express all Henkin quantifiers.

In addition to the usual first-order syntax, we also have *independent quantification*: If ϕ is a formula, x a variable, and W a finite set of variables, then $\forall x/W. \phi$ and $\exists x/W. \phi$ are formulae. The intention is that W is the set of independent variables, whose values the player is not allowed to know at this choice point: thus the Henkin quantifier $\begin{smallmatrix}\forall x & \exists y \\ \forall u & \exists v\end{smallmatrix}$ can be written as $\forall x/\emptyset. \exists y/\emptyset. \forall u/\{x, y\}. \exists v/\{x, y\}$. If one then plays the usual model-checking game with this additional condition, which can be formalized by requiring strategies to be uniform in the 'unknown' variables, one gets a game semantics which characterizes the Skolem function semantics in the sense that Eloise has a winning strategy iff the formula is true.

This logic is called by Hintikka 'independence-friendly' logic. Study of this particular formalism has been mostly carried out by Hintikka and colleagues; but there has been over the last thirty years a continued interest in branching quantification in natural language semantics, increased now by the current popularity of 'Game Theoretical Semantics'. (The recent thesis [Pie00] contains a most useful account of this area.) However, there has been little interest in the computer science temporal logic community.

3 Independence-friendly modal logic

One reason for this is that at first sight, independence-friendly modal logic makes little sense. Suppose that we extend the usual syntax of modal logic with the Hintikka slash;

we will also need to assign a tag to each modality, so that we can refer back to it after a slash.

Definition 1 The syntax of independence-friendly modal logic (IFML) is given as follows. Let $\alpha, \beta, \ldots$ range over a countable set of *tags*, $a, b, \ldots$ over a set of *labels*. tt and ff are IFML formulae. If Φ_1 and Φ_2 are IFML formulae, so are $\Phi_1 \vee \Phi_2$ and $\Phi_1 \wedge \Phi_2$; and so are $\langle a \rangle_{\alpha/\beta_1,\ldots,\beta_m} \Phi_1$ and $[a]_{\alpha/\beta_1,\ldots,\beta_m} \Phi_1$.

Note that we have not included negation in the language. This is intentional, since in IF logics negation is not a simple concept: as we shall see below, IF logics are essentially three-valued. A full discussion of negation in IF logics is given by [Hod97]. In this article, we are not considering languages with either classical negation or 'game negation'.

Certain syntactic conditions may be imposed on IFML formulae:

Definition 2 An IFML formula Φ is *well-formed* if

(a) in every subformula $\langle a \rangle_{\alpha/\beta_1,\ldots,\beta_m} \Psi$, the *bound tag* α is uniquely bound in Φ;

(b) every *independent tag* β_i *of* α is bound in some higher modality in Φ.

It is moreover *good* if

(c) the *dependency relation* on tags given by $\alpha \prec \beta$ if β is not an independent tag of α, is transitive.

We will for this paper restrict ourselves to good formulae.

Of the well-formedness requirements, (a) is a convenience to avoid renaming, but (b) is more controversial: it implies, for example, that a subformula of a well-formed formula is not in general well-formed. This is an issue related to questions of compositional semantics; see [Hod97] for a discussion.

The 'goodness' requirement is a restriction largely for technical convenience. If the dependency relation is not transitive, one can have a phenomenon called 'signalling' [Hod97], whereby intendedly independent choices can be made dependent. Although this is interesting in certain linguistic applications, in 'normal' mathematics, and arguably in logics for concurrency, it is undesirable.

Obviously, the intended semantics of an independence-friendly modal logic is that the existential choice in the $\langle a \rangle_{\alpha/\beta_1,\ldots,\beta_m}$ must be made independently of the choices made in the modalities tagged by β_i. However, in a standard transition system semantics for modal logic, the choices available at a modality are determined by the choices made in earlier modalities, and thus in general it makes no sense to ask for an independent choice.

This problem is removed if the events referred to in the modalities are 'independent' in some sense. For example, in a system comprising two parallel, non-communicating, components, two independent modalities can reasonably refer to choices made in different components. Moreover, the two independent local choices may result in only a single action at a global system level, as when in CCS two actions synchronize; it is this situation that gives the new expressive power in the ATL of [AHK97], and in the 'Henkin modal logic' of [Bra00]. This observation then naturally raises the question of the relationship between independence in the meaning of Hintikka, and independence in semantic models for concurrency.

To examine this question, we shall revert from models with independence implicitly given by locality, to a model with explicit independence. Of the many possibilities, let us choose *transition systems with independence*; these are perhaps the nearest model to ordinary labelled transition systems, and have been used by Nielsen and others to study branching-time logics of (concurrent) independence.

First, we banish a confusing clash of terminology. In 'transition systems with independence', the independence is concurrency, in the model; we wish to relate this to Hintikka-style logical 'independence'. Therefore, *henceforth, concurrent model independence will be called 'concurrency'; 'independence' will be used only to refer to logical independence*. We stress that 'concurrency' is here being used as an *ad hoc* term to distinguish model independence from logic independence. In the literature, 'concurrency' is a distinct concept from model 'independence'; because we will make restrictions on our classes of models, the distinction does not occur in our setting. (We welcome suggestions for better terminology.)

Definition 3 A *coherent transition system with concurrency* (TSC) is a labelled transition system with states S, labels L, and transition relation $\rightarrow \subseteq S \times L \times S$, together with a relation $\mathcal{C} \subseteq \rightarrow \times \rightarrow$ and an initial state s_0. Two transitions $t_1 = (s_1 \xrightarrow{a_1} s_1')$ and $t_2 = (s_2 \xrightarrow{a_2} s_2')$ are *concurrent* if $(t_1, t_2) \in \mathcal{C}$. A relation $\prec$ between transitions with the same label is defined by

$$s_1 \xrightarrow{a} s_1' \prec s_2 \xrightarrow{a} s_2' \Leftrightarrow \exists b.\, (s_1' \xrightarrow{b} s_2')\ \mathcal{C}\ (s_1 \xrightarrow{a} s_1')\ \mathcal{C}\ (s_1 \xrightarrow{b} s_2)\ \mathcal{C}\ (s_2 \xrightarrow{a} s_2')$$

(i.e., the two a-transitions form a diamond with two b-transitions independent of a; notionally, the two a-transitions are the same a-'event', and the two b-transitions are the same b-'event'); $\sim$ is the reflexive, symmetric and transitive closure of $\prec$, and it groups transitions into *events*. In addition, the relation $\mathcal{C}$ is required to satisfy four natural axioms which ensure that an event has a unique outcome at a given state, that concurrent transitions may occur in either order, that concurrency respects events, and that two concurrent events can occur one after the other:

1. $s \xrightarrow{a} s_1 \sim s \xrightarrow{a} s_2 \Rightarrow s_1 = s_2$
2. $s \xrightarrow{a} s_1\ \mathcal{C}\ s_1 \xrightarrow{b} u \Rightarrow \exists s_2.\, s \xrightarrow{a} s_1\ \mathcal{C}\ s \xrightarrow{b} s_2\ \mathcal{C}\ s_2 \xrightarrow{a} u$
3. $s \xrightarrow{a} s_1 \prec s_2 \xrightarrow{a} u\ \mathcal{C}\ w \xrightarrow{b} w' \Rightarrow s \xrightarrow{a} s_1\ \mathcal{C}\ w \xrightarrow{b} w'$

   and $w \xrightarrow{b} w'\ \mathcal{C}\ s \xrightarrow{a} s_1 \prec s_2 \xrightarrow{a} u \Rightarrow w \xrightarrow{b} w'\ \mathcal{C}\ s_2 \xrightarrow{a} u$
4. $s \xrightarrow{a} s_1\ \mathcal{C}\ s \xrightarrow{b} s_2 \Rightarrow \exists u.\, s_1 \xrightarrow{b} u\ \mathcal{C}\ s \xrightarrow{a} s_1$

(a plain TSC need not satisfy axiom 4, the coherence axiom; however, most reasonable models and classes of models are coherent, and we need it for Theorem 17, so we adopt it as a standard requirement). Consequently, a firing sequence of transitions gives rise to a partial order of events, which can be linearized into several different transition sequences, in the usual way of partial order semantics. (Note: in the literature, I is used rather than $\mathcal{C}$, as TSCs are called TSIs.)

In graphical depictions of TSCs, concurrent transitions are denoted by putting the symbol $\mathcal{C}$ inside the commutative square, and the initial state is marked by a circle (when it is not obvious).

We can now define a semantics for IFML, given *à la* Hintikka, by defining its model-checking game as a game of imperfect information. A consequence of this is that the semantics is not defined on states, but requires some history to be kept.

Definition 4 A *tagged run* of a TSC is a sequence $s_0 \xrightarrow[\alpha_0]{a_0} \dots \xrightarrow[\alpha_{n-1}]{a_{n-1}} s_n$, where the α_i are distinct *tags*; we shall also use the tag α_i to refer to the transition $s_i \xrightarrow{a_i} s_{i+1}$. We let ρ, σ etc. range over tagged runs, and use obvious notations for extensions of runs.

A *position* of the model-checking game for an IFML formula Φ on a TSC is a pair of a tagged run and a subformula, written $\rho \vdash \Psi$.

The initial position is $s_0 \vdash \Phi$.

The rules of the game are as follows:

- At a position $\rho \vdash \mathrm{tt}$, Eloise wins; at $\rho \vdash \mathrm{ff}$, Abelard wins.
- At $\rho \vdash \Phi_1 \vee \Phi_2$ (resp. $\vdash \Phi_1 \wedge \Phi_2$), Eloise (resp. Abelard) chooses a new position $\rho \vdash \Phi_i$.
- At $\rho = s_0 \xrightarrow[\alpha_0]{a_0} \dots \xrightarrow[\alpha_{n-1}]{a_{n-1}} s_n \vdash \langle b \rangle_{\beta/\alpha_{i_1},\alpha_{i_2},\dots,\alpha_{i_m}} \Psi$ (resp. $\vdash [b]_{\beta/\alpha_{i_1},\alpha_{i_2},\dots,\alpha_{i_m}} \Psi$), Eloise (resp. Abelard) chooses a transition $s_n \xrightarrow{b} t$ that is concurrent with all the transitions $s_{i_j} \xrightarrow{a_{i_j}} s_{i_j+1}$, and the new position is $\rho \xrightarrow[\beta]{b} t \vdash \Psi$.

Tags are, of course, merely syntactic sugar; it suffices to identify the ith transition by i. However, tags are convenient to match the definition of IFML.

As usual, a strategy for Eloise is a function from her positions to choices. Imperfect information games are handled by imposing additional conditions on strategies.

Definition 5 An Eloise strategy f is *uniform* if the choice at a $\langle\rangle$ position is uniform in the specified independent earlier choices, in the following sense:

Let $\rho \vdash \langle b \rangle_{\beta/\alpha_{i_1},\alpha_{i_2},\dots,\alpha_{i_m}} \Psi$ be as above. The strategy f must choose $s_n \xrightarrow{b} t$ such that if $s_0 = s'_0 \xrightarrow{a_0} \dots \xrightarrow{a_{n-1}} s'_n \vdash \langle b \rangle_{\beta/\alpha_{i_1},\alpha_{i_2},\dots,\alpha_{i_m}} \Psi$ is any other position such that $j \notin \{i_1, \dots, i_m\} \Rightarrow (s_j \xrightarrow{a_j} s_{j+1}) \sim (s'_j \xrightarrow{a_j} s'_{j+1})$, then f chooses a transition $s'_n \xrightarrow{b} t' \sim s_n \xrightarrow{b} t$. (In words, f must choose the same event regardless of the events chosen in the independent modalities. If no such event can be chosen, there is no uniform strategy.) Abelard's uniform strategies are defined similarly.

Definition 6 An IFML formula Φ is *true* in a given TSC, written $s_0 \vDash \Phi$, iff Eloise has a uniform winning strategy for the model-checking game $s_0 \vdash \Phi$.

Φ is *false* iff Abelard has a uniform winning strategy.

Φ is *determined* iff it is either false or true.

The non-determinacy of the model-checking game is a characteristic feature of independence-friendly logic. For a simple example, consider the TSC generated by the CCS process $((a.c + a.\overline{c}) \mid (b.c + b.\overline{c}))\backslash c$ (in which the a-transitions are independent of the b-transitions), and the formula $[a]_\alpha \langle b \rangle_{\beta/\alpha} \langle \tau \rangle \mathrm{tt}$. This formula is not true, since Eloise cannot choose a b-transition so as to synchronize unless she knows which a-transition was chosen;

but it is also not false, since Abelard has no strategy for falsifying it. For practical purposes, we may consider non-truth to be falsehood.

4 IFML equivalence

One of the first questions about any logic is, what is the induced equivalence? In the case of IFML (or indeed the simpler Henkin modal logic of [Bra00]), the definition of equivalence itself is problematic, because of the non-determinacy. We take the weaker (practical) definition, and say

Definition 7 Two TSCs S and T are IFML-equivalent, $S \sim_{\mathrm{IFML}} T$, if for every IFML formula Φ, $S \models \Phi \Leftrightarrow T \models \Phi$.

Logically induced equivalences are typically characterized by a game naturally related to the satisfaction game: for modal logic, we have bisimulation games and model-checking games, for first-order logic we have Ehrenfeucht–Fraïssé games and Hintikka games. For IF logics, the outscoping nature of the / makes such a formulation harder, and to our knowledge the only published E–F game [Vää00] for IF logics is explicitly higher-order. We will consider E–F games for independence logics in a later article; here we study IFML equivalence by relating them to known equivalences in true concurrency.

5 Restrictions on models

For the remainder of this paper, we will consider restricted classes of models. Analysing the effect of removing the restrictions is left to later work.

Firstly, *all TSCs will be image-finite*: that is, for any state s and label a, there are only finitely many a-successors of s. This is a standard restriction required to obtain an exact match between finitary modal logic and bisimulation.

Secondly, *all TSCs will be acyclic*: that is, no state is reachable from itself. This restriction avoids the necessity of distinguishing between models and their unfoldings, which in turn avoids the necessity to distinguish multiple occurrences of the 'same' event.

Finally, we require the dependency relations in the models to be transitive. On acyclic models this is formally, but not actually, a further restriction, since events that are formally concurrent but actually causally dependent can be made formally non-concurrent without change to the model's behaviour.

6 Equivalences for concurrency

There are numerous equivalences for concurrency, but there is one spectrum of particularly natural equivalences that appears promising: the spectrum from bisimulation through to coherent hereditary history-preserving bisimulation. These equivalences have several characterizations; we will define them in the style of classical bisimulation, and also give the game characterizations, which will be useful in our results.

The weakest equivalence is ordinary 'strong bisimulation'; this is well known to be too weak for true concurrent properties, but we define it just to help clarify the other definitions. In particular, we will define it on runs, rather than states.

Definition 8 A relation R on pairs of runs of two TSCs S and T is a *(strong) bisimulation* if

A $(s_0, t_0) \in R$

B if $(\sigma, \tau) \in R$ and $\sigma' = \sigma \xrightarrow{a} s$ is a run, then there is t such that $\tau' = \tau \xrightarrow{a} t$ and $(\sigma', \tau') \in R$; and symmetrically if $\tau'' = \tau \xrightarrow{a} t''$ is a run, then there is s'' such that $\sigma \xrightarrow{a} s''$ and $(\sigma'', \tau'') \in R$.

Systems S and T are *(strongly) bisimilar*, $S \sim_{\mathrm{b}} T$, if there is a strong bisimulation between them.

Bisimulation makes no use of the history of a run, and ignores the concurrency, and thus is definable on states of the TSCs, as is usually done. The definition can also be cast in game-theoretic terms:

Definition 9 The *bisimulation game* played between Duplicator and Spoiler on two TSCs S and T is played as follows. Positions are pairs $(\sigma = s_0 \xrightarrow[\alpha_0]{a_0} \dots s_n, \tau = t_0 \xrightarrow[\beta_0]{a_0} \dots t_n)$ of runs from S and T. The initial position is (s_0, t_0). The two players alternate, with Spoiler starting. The rules are:

I Spoiler chooses one of S or T, say S, and chooses a transition $s_n \xrightarrow{a_n} s_{n+1}$. Duplicator must respond in the other system with a transition $t_n \xrightarrow{a_n} t_{n+1}$ extending τ, or else she loses.

II If either player cannot move, the other wins; if play continues for ever, Duplicator wins.

S and T are bisimilar iff Duplicator has a winning strategy for the bisimulation game iff Duplicator has a history-free winning strategy.

Since modal logic characterizes bisimulation, and IFML includes modal logic, it is immediate that $\sim_{\mathrm{IFML}}$ implies $\sim_{\mathrm{b}}$.

A stronger notion of equivalence is obtained [GG89,RT88] by requiring the equivalence to preserve the concurrency relation between matching events. The following formulation is not the original definition, but is equivalent in our framework:

Definition 10 R is a *history-preserving bisimulation (hpb)* if

A $(s_0, t_0) \in R$

C if $(\sigma, \tau) \in R$ and $\sigma' = \sigma \xrightarrow{a} s$ is a run, then there is t such that $\tau' = \tau \xrightarrow{a} t$, and transitions i and j in σ' are concurrent iff transitions i and j in τ' are concurrent, and $(\sigma', \tau') \in R$; and symmetrically.

we write $S \sim_{\mathrm{hpb}} T$ if there is an hpb between S and T.

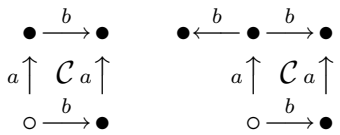
In the characterizing game for hpb Duplicator's choice of transition must have the same pattern of concurrency relations with previous transitions as Spoiler's choice.

Hpb detects at least some true concurrent features: for example, it distinguishes $a.b + b.a$ from $a|b$. However, it has been argued [FrH99,Frö02] that hpb and similar relations such as local/global cause equivalence are really about causality, not about concurrency, and that true concurrency is more correctly captured by stronger equivalences.

The first, initially discouraging, result is that hpb can make distinctions that IFML cannot.

Theorem 11 $\sim_{\mathrm{IFML}} \not\subseteq \sim_{\mathrm{hpb}}$

Proof. Consider the following systems:



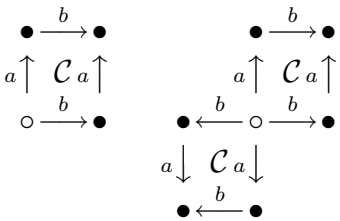
These systems are not hpb, but it may be verified by exhaustive checking that no IFML formula distinguishes them. □

This example will suggest later a possible modification to the definition of IFML; for the present, we continue with the investigation.

It would be surprising if hpb were finer than IFML-equivalence, and indeed it is not.

Theorem 12 $\sim_{\mathrm{hpb}} \not\subseteq \sim_{\mathrm{IFML}}$

Proof. The following counter-example shows the very strong distinguishing power of IFML with respect to conflicting events.



These systems are history-preserving bisimilar, but they are distinguished by the IFML formula $[b]_\beta\langle a\rangle_{\alpha/\beta}\mathrm{tt}$, which is true in the left system, but not in the right, since in the right hand system the choice of b's determines the choice of a's and vice versa. □

A stronger equivalence from concurrency theory is *hereditary (or strong) history-preserving bisimulation (hhpb)* [Bed91,JNW96]. Its relational characterization is

Definition 13 R is a *hereditary history-preserving bisimulation (hhpb)* if

A $(s_0, t_0) \in R$

B if $(\sigma, \tau) \in R$ and $\sigma' = \sigma \xrightarrow{a} s$ is a run, then there is t such that $\tau' = \tau \xrightarrow{a} t$ and $(\sigma', \tau') \in R$; and symmetrically;

D if $(\sigma = s_0 \xrightarrow[\alpha_0]{a_0} \ldots s_n, \tau = t_0 \xrightarrow[\beta_0]{a_0} \ldots t_n) \in R$, and transition α_i is *backwards enabled* in σ, meaning that α_i is concurrent with every later α_j, then β_i is backwards enabled in τ and $(\sigma', \tau') \in R$, where σ' is obtained from σ by using the TSC diamond axioms to push α_i to the end, and then deleting α_i, and similarly τ' is obtained from τ by likewise 'backtracking' β_i; and symmetrically.

The rather complex looking clause D is nothing more than undoing the latest action in some concurrent component; viewing a run as a partial order, rather than a sequence, it is simply the deletion of a maximal element.

It is easy to see that clauses B and D imply that hhpb also satisfies clause C of the hpb definition, and so hhpb is finer (and indeed strictly finer) than hpb. The natural game characterization [NiC94] of hhpb is

Definition 14 The *hhpb* game played between Duplicator and Spoiler on two TSCs S and T is played as follows. Positions are pairs $(\sigma = s_0 \xrightarrow[\alpha_0]{a_0} \dots s_n, \tau = t_0 \xrightarrow[\beta_0]{a_0} \dots t_n)$ of runs from S and T. The initial position is (s_0, t_0). The two players alternate, with Spoiler starting. Spoiler may move in two ways, to which Duplicator must respond.

(1) Spoiler chooses one of S or T, say S, and chooses a transition $s_n \xrightarrow{a_n} s_{n+1}$. Duplicator must respond in the other system with a transition $t_n \xrightarrow{a_n} t_{n+1}$ extending τ, or else she loses.

(2) Alternatively, Spoiler chooses S or T (say S), and a transition $s_i \xrightarrow{a_i} s_{i+1}$ in σ which is backward-enabled. He then 'backtracks' along this transition, as in the relational definition. Duplicator must then respond by backtracking the ith transition in the other system; if this transition is not backwards enabled, she cannot move.

(3) If either player cannot move, the other wins; if play continues for ever, Duplicator wins.

Hhpb looks like a good candidate for comparison with IFML. For the same reasons as hpb, hhpb can distinguish systems that IFML cannot; but one might wonder whether hhpb is finer than IFML-equivalence (for our restricted models). However, this is not the case:

Theorem 15 $\sim_{\text{hhpb}} \not\subseteq \sim_{\text{IFML}}$

Proof. The two systems of Theorem 12 are also hereditary history-preserving bisimilar. □

To find a sufficiently fine equivalence in the field of concurrency, we need to impose *coherence* requirements. These are requirements similar to the coherence axiom for TSCs, forcing the equivalence to respect, to varying degrees, the possibility for concurrently enabled events to occur consecutively in either order. Cheng [Che96] briefly studied what he called *strong coherent history-preserving bisimulation*, which is obtained by adding a coherence requirement to hhpb; we call it *coherent hereditary history-preserving bisimulation (chhpb)*. However, the coherence requirement there is still too weak; we use a somewhat stronger requirement, that we call *strict coherence*.

Firstly, we introduce some notation to save space in the following. We will describe runs of a system in terms of their events, denoted by greek letters: so the run $s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} \dots$ will be written $\alpha_0\alpha_1\dots$ (recall that a run of a TSC is uniquely determined by its start state (assumed to be the initial state) and its events). The scope for confusion with tags is intentional.

Definition 16 R is a *strictly coherent hereditary history-preserving bisimulation (schhpb)* if

The clauses of hhpb, together with

E if $(\sigma\alpha, \tau\alpha') \in R$ and $(\sigma\beta, \tau\beta') \in R$ and $\alpha \mathcal{C} \beta$, then $\alpha' \mathcal{C} \beta'$ and $(\sigma\alpha\beta, \tau\alpha'\beta') \in R$, and symmetrically.

Theorem 17 *If S and T are schhpb, then they are IFML-equivalent.*

Proof. To prove this theorem, we start with lemmas that establish the existence of 'uniform' choices of events. We assume the conditions of the theorem, and let R be an schhpb between S and T.

Lemma 18 Let $(\sigma\alpha, \tau\alpha') \in R$. Let β with label b be the last event in σ concurrent with α. Since R is an hhpb, there is a matching event β' in τ concurrent with α'. By the transitivity of dependence, β, β' are backwards-enabled, so let σ_1, τ_1 be the result of backtracking β, β' in σ, τ. Let β_1 be any other b-labelled event concurrent with α and all events after β in σ that is enabled at σ_1. Then

(1) Let σ_2 be the run obtained by substituting β_1 for β in σ. σ_2 and $\sigma_2\alpha$ are valid runs.
(2) If β_1' is a b-labelled event such that τ_2 formed by substituting β_1' for β' in τ is such that $(\sigma_2, \tau_2) \in R$, then β_1' is concurrent with α' and $(\sigma_2\alpha, \tau_2\alpha') \in R$.

Proof. α is enabled at σ_1, β_1 is enabled at σ_1, so by the TSC coherence axiom, $\sigma_1\beta_1\alpha$ is a run. Since β_1 is concurrent with the events after β in σ, it can be commuted with them to give $\sigma_2\alpha$ as a run.

Now since $(\sigma_2, \tau_2) \in R$, β_1' is concurrent with all the subsequent events in τ_2 (by the history-preserving property of R). Therefore by commutativity of concurrent actions and the hereditary and coherent properties of R, we can push β_1, β_1' to the end to give $(\sigma_1\beta_1, \tau_1\beta_1') \in R$. We also have $(\sigma_1\alpha, \tau_1\alpha') \in R$, and $\alpha \mathcal{C} \beta_1$. By the strict coherence property of R we have that $\alpha' \mathcal{C} \beta_1'$ and $(\sigma_1\beta_1\alpha, \tau_1\beta_1'\alpha') \in R$; now by commuting β_1, β_1' back again, we obtain $(\sigma_2\alpha, \tau_2\alpha') \in R$.

A stronger version of this lemma is

Lemma 19 If $(\sigma\alpha, \tau\alpha') \in R$, and if $(\sigma_2, \tau_2) \in R$ is such that σ_2 is the result of substituting events $\gamma_1, \ldots, \gamma_k$ concurrent with α for the events $\beta_1, \ldots, \beta_k$ concurrent with α in σ, and τ_2 is the result of substituting similarly labelled events $\gamma_1', \ldots, \gamma_k'$ for the corresponding events $\beta_1', \ldots, \beta_k'$ of τ, then $(\sigma_2\alpha, \tau_2\alpha') \in R$.

Proof. First backtrack the events $\beta_2, \ldots, \beta_k$ and $\beta_2', \ldots, \beta_k'$ to give $(\sigma_1\alpha, \tau_1\alpha') \in R$. (Note that by the transitivity of dependence, any event dependent on any β_j is concurrent with α and therefore is some $\beta_{j'}$). Apply Lemma 18 to get $(\sigma_3\alpha, \tau_3\alpha') \in R$, where σ_3 is the substitution of γ_1 for β_1 in σ_1, and similarly for τ_3. Because $(\sigma_2, \tau_2) \in R$, by backtracking we have that $\gamma_2, \ldots, \gamma_k$ and $\gamma_2', \ldots, \gamma_k'$ are enabled at (σ_3, τ_3); then by the coherent and hereditary properties again we can fire these events and commute them to obtain $(\sigma_2\alpha, \tau_2\alpha') \in R$.

We now continue with the proof of the theorem.

Let Φ be an IFML formula such that $S \vDash \Phi$. We shall use the schhpb relation and Eloise's winning uniform strategy for $S \vdash \Phi$ to allow her to win $T \vdash \Phi$. To do this, we shall construct a T-strategy for Eloise by mirroring S-moves onto T in such a way that (a) the S and T histories are schhpb, and (b) the resulting constructed T-strategy is winning and uniform.

Suppose that in the model-checking games we have reached positions $\sigma \vdash \Psi$ and $\tau \vdash \Psi$, with $(\sigma, \tau) \in R$. First, we consider the case where Abelard is to move in the T game. At

a conjunction, we directly copy Abelard's T move to the S game. The interesting case is where $\Psi = [a]_{\alpha/\beta,\ldots}\Psi'$. Suppose Abelard moves by extending τ to $\tau\alpha'$. Eloise mirrors this move in S by some α such that $(\sigma\alpha, \tau\alpha') \in R$. Eloise may have a choice of α; by Lemma 19 (applied in the reverse direction), the choice need depend only on the events in τ that are not concurrent with α', since any given choice works also for any run obtained by changing concurrent events in τ. If Abelard cannot move, then Eloise has won the play.

Now consider the case where Eloise is to move: we must define her strategy. In the case of a disjunction, she makes the same move in the T game that she makes in the S game. Again, the interesting case is $\Psi = \langle a\rangle_{\alpha/\beta,\ldots}\Psi'$. In the S game, her strategy chooses some α; her T-strategy choice will be some event α' such that $(\sigma\alpha, \tau\alpha') \in R$. Again by Lemma 19, she can choose α' to depend only on the events not concurrent with α. Since in S her choice of α is by assumption uniform in the concurrent events $\beta, \ldots$, and every Abelard event in σ has been chosen uniformly in its concurrent events, her choice of α' is uniform in $\beta', \ldots$. (Recall that in the logic as well as the model the dependency relation is required to be transitive.)

The possibility of Eloise being unable to move does not arise, since by assumption we are following a winning strategy in the S game.

Thus we have a strategy for Eloise that is winning, and by construction is uniform in the independent events at each diamond modality, so $T \vDash \Phi$. □

7 Restricting conflict

So far, the results we have obtained give a somewhat confusing picture of the relationship between independence and concurrency. Although all but the strongest concurrent equivalence we have examined are incomparable with $\sim_{\mathrm{IFML}}$, there is still an obvious, but not yet clear, intuitive link between them. In the study of concurrent equivalences, one technique for understanding the differences between equivalences is to study them on restricted classes of models, to determine which model properties are really detected by the equivalences. The second author's thesis [Frö02] includes a study of free-choice Petri nets (in which a choice between two events may not be influenced by any other concurrent event). Free-choice systems are a class with many attractive properties, and intuitively they seem not to have the properties that are exploited by hereditary equivalences. This led Cheng to conjecture [Che96] that hhpb and hpb coincide on free-choice systems. Fröschle disproved this conjecture [Frö02], and the counter-examples she used are IFML-equivalent; whereas the counter-examples of Theorem 12 are not free-choice systems. This led us to wonder if $\sim_{\mathrm{IFML}}$ might be substantially less powerful on free-choice systems: for example, it might be coarser than hhpb, or even coarser than hpb. The latter is, however, not true, although now the counter-example is much more complex, and seems to need the use of a full Henkin-style modality rather than just the simple $[]\langle\rangle$ that sufficed before.

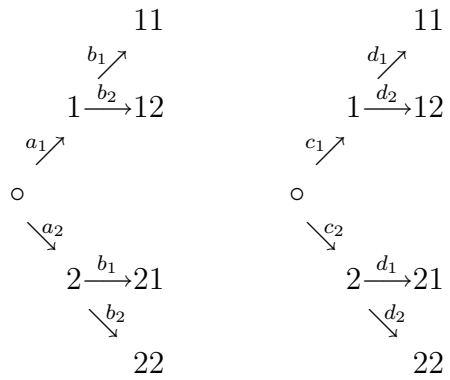
'Free-choice' is technically a property of Petri nets, and thus assumes a built-in notion of locality. The following definition is an adaptation of this notion for TSC.

Definition 20 A TSC is *free-choice* if whenever $s \xrightarrow{a_1} s_1$ and $s \xrightarrow{a_2} s_2$ are two non-concurrent transitions, and $s \xrightarrow{b} t$ is a transition concurrent with either of them, then $t \xrightarrow{a_1} t_1 \sim s \xrightarrow{a_1} s_1$ and $t \xrightarrow{a_2} t_2 \sim s \xrightarrow{a_2} s_2$ for some t_1, t_2. In words, a non-deterministic choice cannot be restricted by the execution of a concurrent event.

Theorem 21 $\sim_{\text{hpb}} \not\subseteq \sim_{\text{IFML}}$ *even on free-choice systems.*

Proof. The counter-example we have at present is rather complex to draw in full, so we will give a combined graphical and syntactic description. The two systems we define are free-choice in the sense above, and they can also be generated from two free-choice Petri nets; however, the Petri net versions are even more complex to describe, so we will just present the TSCs.

Let A and C be the two systems



and let P be their concurrent composition, which is a pyramid with 16 distinct final states on the square face. The systems S and T are formed by adding an e-transition to some of these final states, as indicated by the following matrix in which the columns are the A states $11, 12, 21, 22$, the rows are the C states $11, 12, 21, 22$, and the entries indicate the presence of an e-transition in the given systems.

$$\begin{array}{cccc} - & ST & - & S \\ ST & - & S & T \\ S & T & ST & - \\ - & S & - & ST \end{array}$$

It may be verified (and has been checked with the Edinburgh Concurrency Workbench!) that S and T are strongly bisimilar, and since the concurrency relations are the same, they are also history-preserving bisimilar. However, the following IFML formula is true of S but not of T:

$$[a]_\alpha \langle b \rangle_\beta [c]_{\gamma/\alpha\beta} \langle d \rangle_{\delta/\alpha\beta} \langle e \rangle \mathrm{tt}.$$

(This is because in S, Eloise can choose b_1 after Abelard's a_1 and b_1 after Abelard's a_2; then she can choose d_2 after c_1 and d_1 after c_2, without depending on a, and she ends up in a state with an e-transition. In T, on the other hand, no such uniform choice of d exists.) □

This counter-example is also interesting because it is another and rather different counter-example to show that hhpb is strictly finer than hpb on free-choice systems, and

the demonstration of this throws some light on the relationship between hhpb and IFML. The two systems above are distinguished by the formula

$$[a]\langle b\rangle[c]\langle d\rangle \textcircled{b}@[a]\langle b\rangle\langle e\rangle \mathrm{tt}$$

of the characteristic logic [NiC94] for hhpb (where @ is the modality of backtracking an a action). In a later article, we shall discuss the nature of the relationship between this formula and the IFML formula.

It is now natural to ask whether hhpb is finer than $\sim_{\mathrm{IFML}}$ on free-choice systems. We suspect that this is true, but have not so far been able to prove it:

Conjecture 22 $\sim_{\mathrm{hhpb}} \subseteq \sim_{\mathrm{IFML}}$ *on free-choice systems.*

8 A variation on IFML

An alternative approach to restricting the models is to consider alternative semantics for the logic: although our IFML semantics seems the natural definition, perhaps it isn't. In particular, the fact that all the concurrent equivalences (apart from bisimulation itself) distinguish systems that IFML does not, is unsatisfactory. Upon inspection of the counter-example of Theorem 11, one can see that this is due to a rather simple mismatch between the expressivity of the concurrent logics and IFML: the concurrent logics can express 'a followed by a concurrent b', 'a followed by a dependent b', and also 'a followed by choice of concurrent and dependent b'. IFML, on the other hand, can express 'a followed by a concurrent b', and 'a followed by a dependent b and no concurrent b', but cannot distinguish the case where there is a dependent b as well as a concurrent b.

It is possible to make a small change to the semantics of IFML which addresses this issue. Let us call the result IFMLd (IFML with explicit dependence), defined by the following change to the model-checking game of Defn 4:

Definition 23 The IFMLd game is as for IFML except that:

- At $\rho = s_0 \xrightarrow{a_0} \ldots \xrightarrow{a_{n-1}} s_n \vdash \langle b\rangle_{\beta/\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}}\Psi$ (resp. $\vdash [b]_{\beta/\alpha_{i_1},\alpha_{i_2},\ldots,\alpha_{i_m}}\Psi$), Eloise (resp. Abelard) chooses a transition $s_n \xrightarrow{b} t$ that is concurrent with all the transitions α_{i_j} *and not concurrent with any other transition* α_k, and the new position is $\rho \xrightarrow[\beta]{b} t \vdash \Psi$.

That is, choices in modalities are required to be concurrent with previous choices if and only if they are logically independent, rather than just if.

Note that for this modification even to make sense, we need the restriction to transitive dependency relations that we imposed on our models.

This is superficially attractive, and certainly deals with the example of Theorem 11, and we

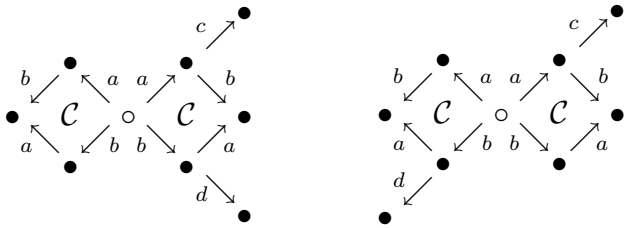
Conjecture 24 $\sim_{\mathrm{IFMLd}} \subseteq \sim_{\mathrm{hpb}}$

but have not established this conjecture.

It is very tempting to conjecture that $\sim_{\mathrm{IFMLd}} \subseteq \sim_{\mathrm{hhpb}}$, but this conjecture fails.

Theorem 25 $\sim_{\mathrm{IFMLd}} \not\subseteq \sim_{\mathrm{hhpb}}$

Proof. The following is a notorious example [NiC94] of two systems that are not hhpb (although they are hpb):



It may be verified by exhaustive (and in this case somewhat exhausting) checking that neither IFML nor IFMLd can distinguish them.

It should, however, be pointed out that despite the naturalness of IFMLd, there are some unpleasant consequences of adopting it. In particular, it becomes impossible to express the ordinary modal logic formula $[a]\langle b\rangle\Phi$, where the choice of b *may* depend on a, if a and b happen to be concurrent. (It is for this reason that Conjecture 24 is not the simple result one would like.) A referee remarks that perhaps one needs yet another definition, expressing on which states a transition depends. This is indeed a possibility; however, the explicit introduction of states involves either adding state variables to the logic (as the referee suggests, producing a kind of hybrid logic), or using models with a notion of locality built in, as is done in [Bra00] and in unpublished work of the first author and colleagues on Petri net logics.

9 Conclusion

We have shown that it is possible to define a modal version of the Hintikka–Sandu independence-friendly logic, and that such a logic naturally requires true concurrent models. We have looked at the relationship between the induced equivalence and the equivalences associated with true concurrent models. The results to date indicate that although there is a natural connection, it is not as clean as one would like. In this article, we have laid the groundwork for a more substantial study. A full analysis will take considerable effort; Fröschle's work on concurrent equivalences has thrown up many subtle points, some of which have an impact on our aim of relating them to independent equivalences.

A particular point of interest is that there are intriguing questions about the exact relationship between backtracking (as used in hhpb), and uniformity (as used in schhpb and in IFML), and we suspect that these questions relate to natural notions of Ehrenfeucht–Fraïssé game for independence logics. (To coin a slogan, the art of independence is in doing second-order things without appearing to do so.) In turn, independence logics may give new insight into the complexity of the concurrent equivalences.

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References

[AHK97] R. Alur, T. Henzinger and O. Kupferman, Alternating-time temporal logic, in *Proc. 38th FOCS (1997)*, 100–109.

[Bed91] M. A. Bednarczyk, Hereditary history preserving bisimulations, manuscript (1991).

[Bra00] J. C. Bradfield, Independence: logics and concurrency, *Proc. CSL 2000*, LNCS **1862** 247–261 (2000).

[Che96] A. Cheng, *Reasoning About Concurrent Computational Systems*, Ph.D. thesis, BRICS DS–96–2, Univ. Aarhus (1996).

[Frö02] S. Fröschle, *Decidability and coincidence of equivalences for concurrency.* Ph.D. thesis, Univ Edinburgh, forthcoming.

[FrH99] S. Fröschle and T. Hildebrandt, On plain and hereditary history-preserving bisimulation. *Proc. MFCS '99*, LNCS **1672**, 354–365 (1999).

[GG89] R. van Glabbeek and U. Goltz, Equivalence notions for concurrent systems and refinement of actions, *Proc. MFCS '89*, LNCS **379** (1989).

[HiS96] J. Hintikka and G. Sandu, A revolution in logic?, *Nordic J. Philos. Logic* **1**(2) 169–183 (1996).

[Hod97] W. Hodges, Compositional semantics for a language of imperfect information, *Int. J. IGPL* **5**(4), 539–563.

[JNW96] A. Joyal, M. Nielsen and G. Winskel, Bisimulation from open maps, *Inform. and Comput.*, **127**(2) 164–185 (1996).

[NiC94] M. Nielsen and C. Clausen, Bisimulations, games and logic, in Karhumaki, Maurer and Rozenberg (eds), *Results and Trends in Theoretical Computer Science: Colloquium in Honour of Arto Salomaa*, LNCS **812** 289-305 (1994). Also as BRICS report RS-94-6.

[Pie00] A. Pietarinen, *Games logic plays. Informational independence in game-theoretic semantics.* D.Phil. thesis, Univ Sussex (2000).

[RT88] A. Rabinovitch and B. Trakhtenbrot, Behaviour structures and nets. *Fund. Inf.* **11**(4) (1988).

[Vää00] J. Väänänen, On the semantics of informational independence, manuscript (2000).